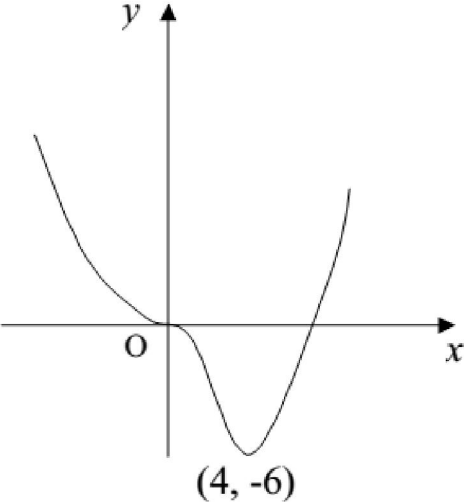
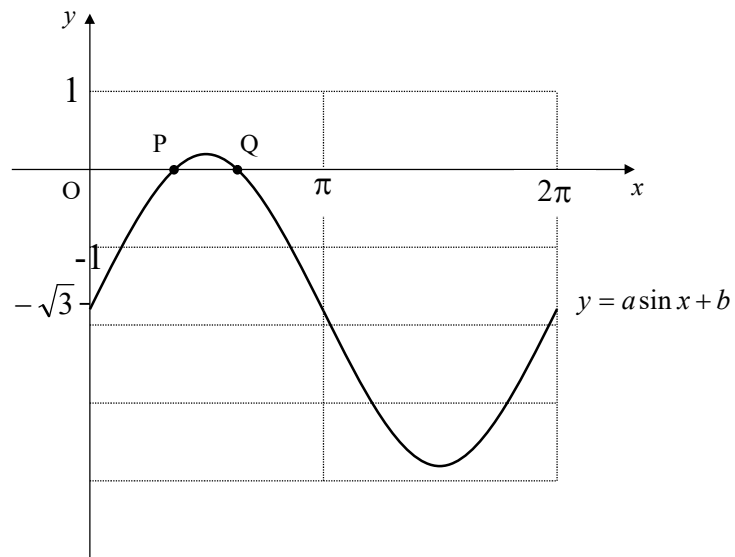


	Higher Prelim Revision 2 – Paper 1 Non-Calculator	30
1.	Find the remainder on dividing the polynomial $x^3 - 2x^2 + 3x - 10$ by $x - 3$	2
2.	Two functions, defined on suitable domains, are given as $f(x) = \frac{1}{x} \quad \text{and} \quad g(x) = x^2 - 1.$ <p>Find an expression for $f(g(x))$, stating any restrictions on the domain.</p>	4
3.	Integrate $\int \frac{6}{x^4} dx$	2
4.	Find the gradient of the straight line parallel to $3x + 2y + 8 = 0$	2
5.	Solve $\log_2(x+1) - 2\log_2 3 = 3$, for x where $x > 0$.	4
6.	Find the equation of the tangent to the curve $y = x^3 - 3x^2 - 4x + 1$ when $x = -1$	4
7.	<div style="display: flex; align-items: center;"> <div style="flex: 1;">  </div> <div style="flex: 2; padding-left: 20px;"> <p>The diagram shows part of the graph of $y = f(x)$.</p> <p>It has stationary points at $(0,0)$ and $(4,-6)$</p> <p>Sketch the graph of the derived function $y = f'(x)$</p> </div> </div>	3
8.	Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p	4

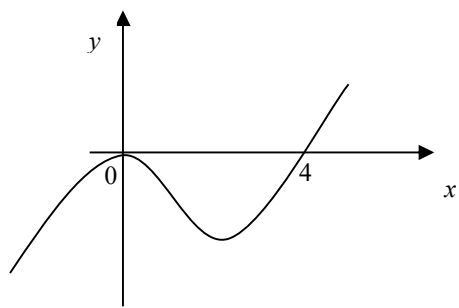
9. The diagram shows the graph of $y = 2 \sin x - \sqrt{3}$ for $0 \leq x \leq 2\pi$.



Determine the exact x -coordinates for the points P and Q.

3

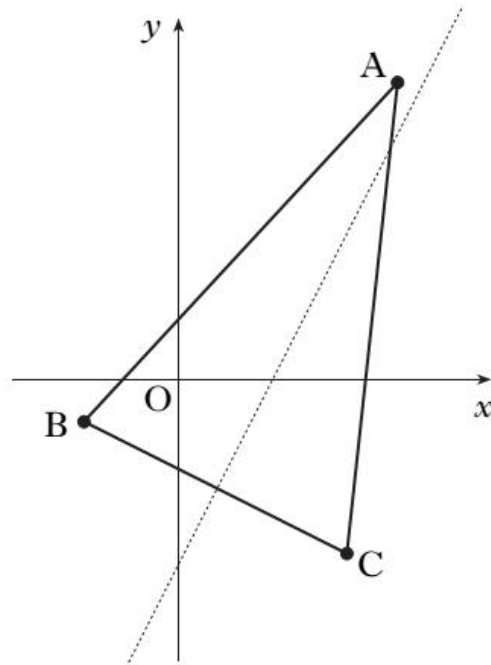
Answers to Paper 1

1. know to use $x = 3$, then work through synthetic division to find a remainder of 8 , or substitute $(3)^3 - 2(3)^2 + 3(3) - 10 = \mathbf{8}$
2. $f(g(x)) = \frac{1}{x^2 - 1}$, restrictions on the domain $x \neq \pm 1$
3. $\frac{6}{x^4} = 6x^{-4}$, $\int 6x^{-4} dx = \frac{6x^{-3}}{-3} + C$, Answer simplifies to $\frac{-2}{x^3} + C$, although this is not necessary for full marks to be awarded
4. $3x + 2y + 8 = 0$ rearranges to $2y = -3x - 8 \rightarrow y = \frac{-3}{2}x - 4$, gradient is $\frac{-3}{2}$
5. Using laws of logs $\log_2 \frac{(x+1)}{3^2} = 3 \rightarrow \frac{(x+1)}{3^2} = 2^3 \rightarrow x + 1 = 72 \rightarrow \mathbf{x = 72}$
6. $\frac{dy}{dx} = 3x^2 - 6x - 4 \rightarrow \frac{dy}{dx}(-1) = 3(-1)^2 - 6(-1) - 4$, gradient is 5 Using point $(-1, 1)$ equation of the tangent is $y - 1 = 5(x + 1)$ or $\mathbf{y = 5x + 6}$
7. Stationary points at $x = 0$ and $x = 4$ become roots of the derived function 
8. For roots to be real, $b^2 - 4ac \geq 0$. $b^2 - 4ac = p^2 - 4(2)(-3)$ $= p^2 + 24$ Any squared term is always ≥ 0 . Therefore $b^2 - 4ac \geq 0$ for all values of p and the roots of the equation are always real
9. $0 = 2 \sin x - \sqrt{3} \rightarrow \frac{\sqrt{3}}{2} = \sin x \rightarrow \theta = \frac{\pi}{3}, 0 = \frac{2\pi}{3}$

1.

The vertices of triangle ABC are $A(7,9)$, $B(-3,-1)$ and $C(5,-5)$

The broken line represents the perpendicular bisector of BC



- (a) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$
- (b) Find the equation of the median from C
- (c) Find the coordinates of the point of intersection between the perpendicular bisector of BC and the median from C

3

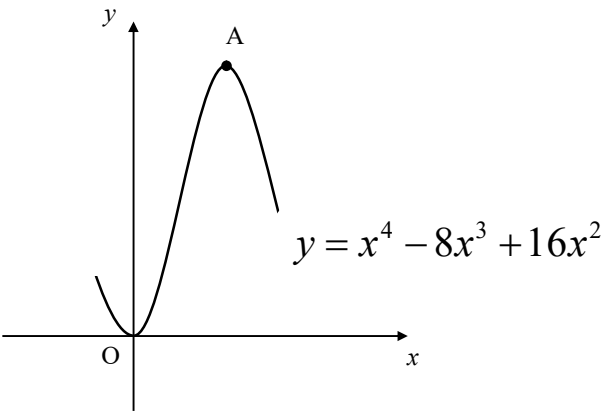
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4

2. The curve $y = 2x^3 - 3x^2 - 3x + 2$ crosses the x -axis at the point $(-1,0)$.

This curve also passes through the x -axis at two further points. Find **algebraically** the coordinates of these points

4

3.	<p>Part of the graph of $y = h(x)$ is shown below.</p>  <p style="text-align: right;">$y = x^4 - 8x^3 + 16x^2$</p> <p>Find the coordinates of point A.</p>	6
4.	<p>Solve algebraically the equation</p> $5 \sin 2x^\circ + 4 \sin x^\circ = 0 \quad \text{for } 0 \leq x < 360$	5
	<p>A radioactive substance decays according to the formula $M_t = M_0 6^{-0.07t}$, where M_0 is the initial mass of the substance and M_t is the mass remaining after t years.</p> <p>Calculate how long a sample of this substance would take to half its original mass.</p>	4
6.	<p>Express $\sqrt{7} \sin x^\circ + 3 \cos x^\circ$ in the form $k \sin(x + \alpha)$</p> <p>Hence solve algebraically the equation</p> $\sqrt{7} \sin x^\circ + 3 \cos x^\circ + 3 = 0 \quad \text{for } 0 \leq x < 360.$	4 3
7.	<p>Evaluate $\int_0^{\frac{\pi}{6}} (x - 2 \sin x) dx$, give your answer correct to 2 decimal places</p>	4

Answers to Paper 2

1(a)	$m_{BC} = \frac{-1}{2} \rightarrow m_{alt} = 2$, midpoint of BC is (1,-3), perpendicular bisector is $y + 3 = 2(x - 1) \rightarrow y = 2x - 5$ as required
(b)	Midpoint of AB is (2,4) median is $y - 4 = -3(x - 2) \rightarrow y = -3x + 10$
(c)	$y = -3x + 10$ and $y = 2x - 5 \rightarrow -3x + 10 = 2x - 5, x = 3 \rightarrow$ P.O.I is (3,1)
2.	for x-intercepts $0 = 2x^3 - 3x^2 - 3x + 2$ $0 = (x+1)(2x^2 - 5x + 2)$ $0 = (x+1)(2x-1)(x-2)$ $x = \frac{1}{2}, x = 2$ (1/2, 0) and (2,0)
3.	$\frac{dy}{dx} = 4x^3 - 24x^2 - 32x$, for stationary points $4x^3 - 24x^2 - 32x = 0$ $4x(x-4)(x-2) = 0$ Stationary points when $x = 0, x = 2$ and $x = 4$ A is (2, 16)
4.	$\sin 2x = 2 \sin x \cos x$ $5(2 \sin x \cos x) + 4 \sin x = 0$ $10 \sin x \cos x + 4 \sin x = 0$ $2 \sin x(5 \cos x + 2) = 0$ $x = 0^\circ, 113.6^\circ, 180^\circ, 246.4^\circ$ $\sin x = 0, \cos x = \frac{-2}{5}$
5.	$5 = 10 \times 6^{-0.07t} \rightarrow \frac{1}{2} = 6^{-0.07t}$ Using log to base 6 $\log_6 \frac{1}{2} = \log_6 6^{-0.07t}$ $\log_6 \frac{1}{2} = -0.07t$ $\frac{\log_6 \frac{1}{2}}{-0.07} = t$ half life is 5.5 years
6.(a)	$k \sin(x + \alpha) = k \sin x \cos \alpha + k \cos x \sin \alpha$ $k \cos \alpha = \sqrt{7}, k \sin \alpha = 3$ $k \sin(x + \alpha) = 4 \sin(x + 48.6)^\circ$
(b)	$4 \sin(x + 48.6)^\circ = -3$ $\sin(x + 48.6)^\circ = \frac{-3}{4}$ $x = 180^\circ, 262.8^\circ$

$$7. \text{ Integrate } \int_0^{\pi/6} (x - 2 \sin x) dx = \left[\frac{x^2}{2} + 2 \cos x \right]_0^{\pi/6}$$

$$\text{Evaluate } \left(\frac{(\pi/6)^2}{2} + 2 \cos(\pi/6) \right) - \left(\frac{0^2}{2} + 2 \cos 0 \right) = -0.13$$

Examples and Extra Practice

	Paper 1			Paper 2	
1.	Synthetic division	Ex 7.18 Pg 153 Q1 Pg 154	1.	Straight lines	Ex 13.11 Pg 301 Ex 13.13 Pg 303 Q4 Pg 305
2.	Composite functions	Ex 4.5 Pg 86 Q5,6 Pg 88	2.	Synthetic Division	Ex 7.22 Pg 156 Q3 Pg 157
3.	Integration	Ex 11.4 Pg 268 Q3 Pg 269	3.	Stationary Points	Ex 10.12 Pg 255 Q3e Pg 257
4.	Straight lines	Ex 13.2 Pg 294 Q3 Pg 295	4.	Solving equations with double angles	Ex 8.15 Pg 189 Q3 Pg 191
5.	Log equations	Ex 1.20 Pg 14 Q1 Pg 13	5.	Exponential half-life	Ex 1.23 Pg 16 Q5 Pg 17
6.	Tangents to curves	Ex 10.2 Pg 239 Q3, Pg 241	6.	Wave function and solving equations with compound angles	Ex 2.33 Pg 51 Ex 8.4 Pg 174 Q1 d,e,f Pg 174
7.	Sketching the derived function	Ex 3.28 Pg 79 Q2 Pg 81	7.	Definite integrals for trig functions	Ex 12.8 Pg 288 Q1 Pg 289
8.	discriminant	Ex 7.31 Pg 168 Q7 Pg 169			
9.	Trig graphs, equations and exact values	Ex 8.3 Pg 173 Q2a,b Pg 175			